# MATH 504 HOMEWORK 4

Due Monday, March 1.

**Problem 1.** *Suppose that $\kappa$ is a regular uncountable cardinal and that $F : \kappa \to \kappa$ is a function. Show that $\{\gamma < \kappa \mid \forall\alpha < \gamma(F(\alpha) < \gamma)\}$ is a club subset of $\kappa$.*

**Problem 2.** *Suppose that $\kappa$ is a regular uncountable cardinal.*

(1) *Suppose that $C$ is a club in $\kappa$ and $S$ is a stationary subset of $\kappa$. Show that $C \cap S$ is also stationary.*
(2) *Suppose $\kappa$ is inaccessible. Show that $\{\tau < \kappa \mid \tau$ is a cardinal$\}$ is a club in $\kappa$.*

**Problem 3.** (1) *Show that for all $\alpha \leq \omega$, $L_\alpha = V_\alpha$.*
(2) *Show that for all $\alpha$, $L_\alpha \subset V_\alpha$ and if $\alpha \geq \omega$, then $|L_\alpha| = |\alpha|$.*
(3) *Show that for all $\alpha \geq \omega$, $L_\alpha \cap Ord = \alpha$ (hint: use induction on $\alpha$).*

**Problem 4.** *Show that if $\kappa$ is a regular uncountable cardinal in $L$, then $L_\kappa$ satisfies all the axioms $ZF\setminus$ Powerset with the exception of Comprehension. (Actually, $L_\kappa$ also satisfies Comprehension, but I will show that in class.)*

**Problem 5.** *Suppose that $M \prec L_{\omega_1}$. Show that $M$ is transitive. (Hint: for $X \in M$, take the $\prec_L$-least onto $f : \omega \to X$. Show that $f$ is definable in $L_{\omega_1}$ from $X$ and use this to show that $f \in M$. Also show $\omega \subset M$. Use these to prove that range of $f$ is a subset of $M$)*